Low-dimensional Embeddings of Logic

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Machine Reading

ACL 2014 Workshop on Semantic Parsing

26th June 2014

Motivation

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Machine Reading and Reasoning

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- "Colugos are arboreal gliding mammals that are found in Southeast Asia."

  $\textsc{mammal}(\textsc{Colugo})$

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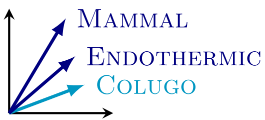

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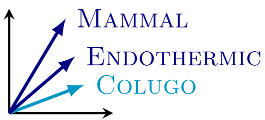

Debugging Distributed Representations

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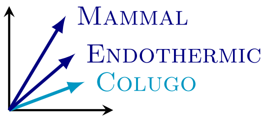

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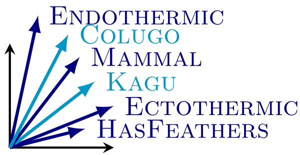

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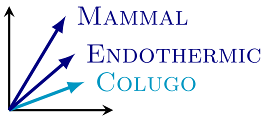

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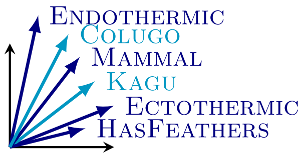

- Wrong Predictions

  $\textsc{mammal}(\textsc{Kagu})$

  $\textsc{ectothermic}(\textsc{Colugo})$

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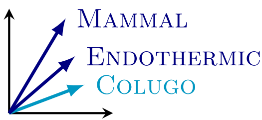

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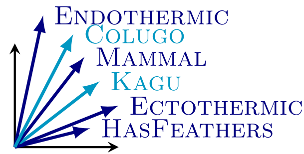

- Wrong Predictions

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I wish I could fix this with...

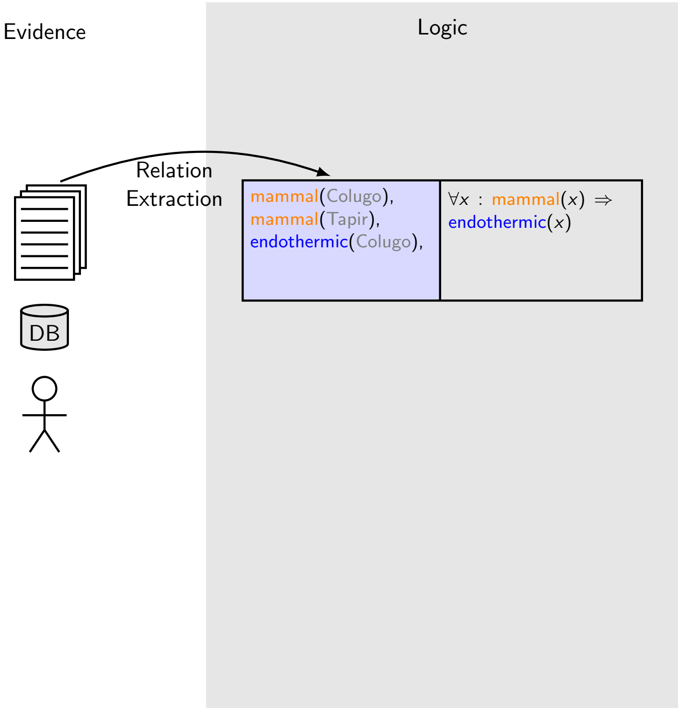
$\forall x : \textsc{hasFeathers}(x) \Rightarrow \neg\textsc{mammal}(x)$

$\forall x : \textsc{animal}(x) \Rightarrow \textsc{endothermic}(x) \oplus \textsc{ectothermic}(x)$

Information Extraction

Evidence

Information Extraction



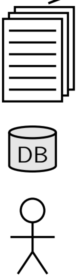
Information Extraction

Evidence

Logic

Discourse Representation Theory or Semantic Parsing

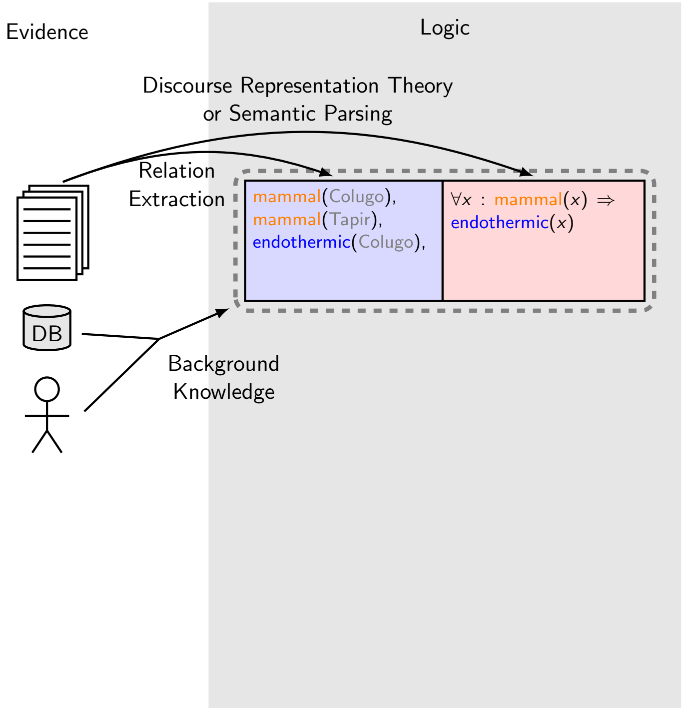
Relation Extraction



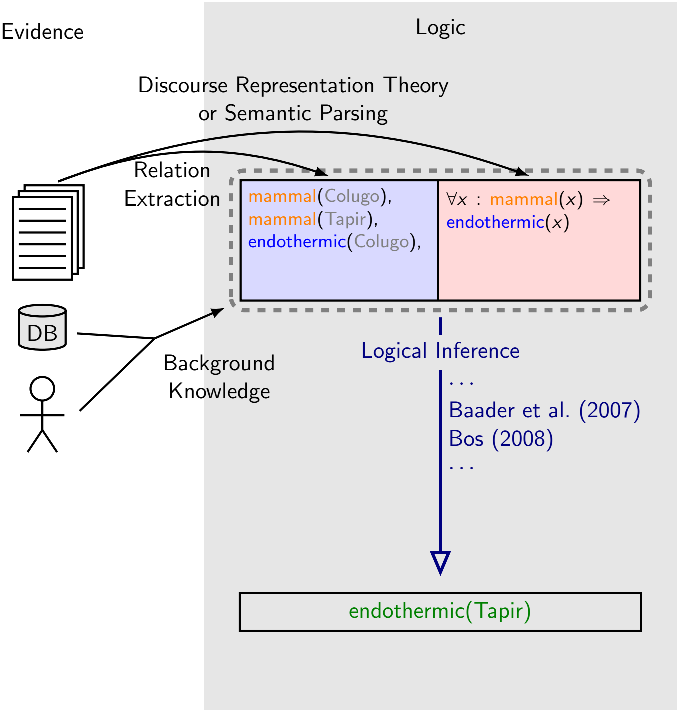
mammal(Colugo),
mammal(Tapir),
endothermic(Colugo),

$\forall x$: mammal(x) $\Rightarrow$ endothermic(x)

Information Extraction



Logical Inference



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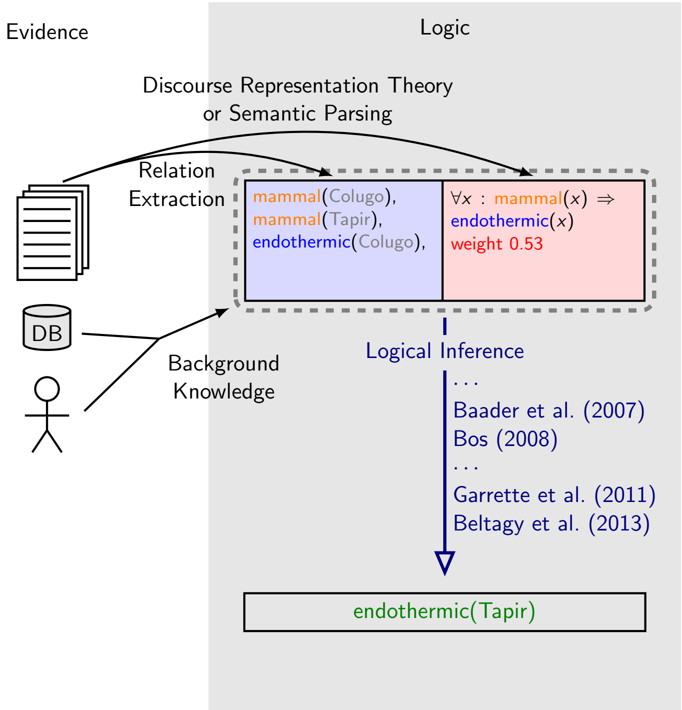
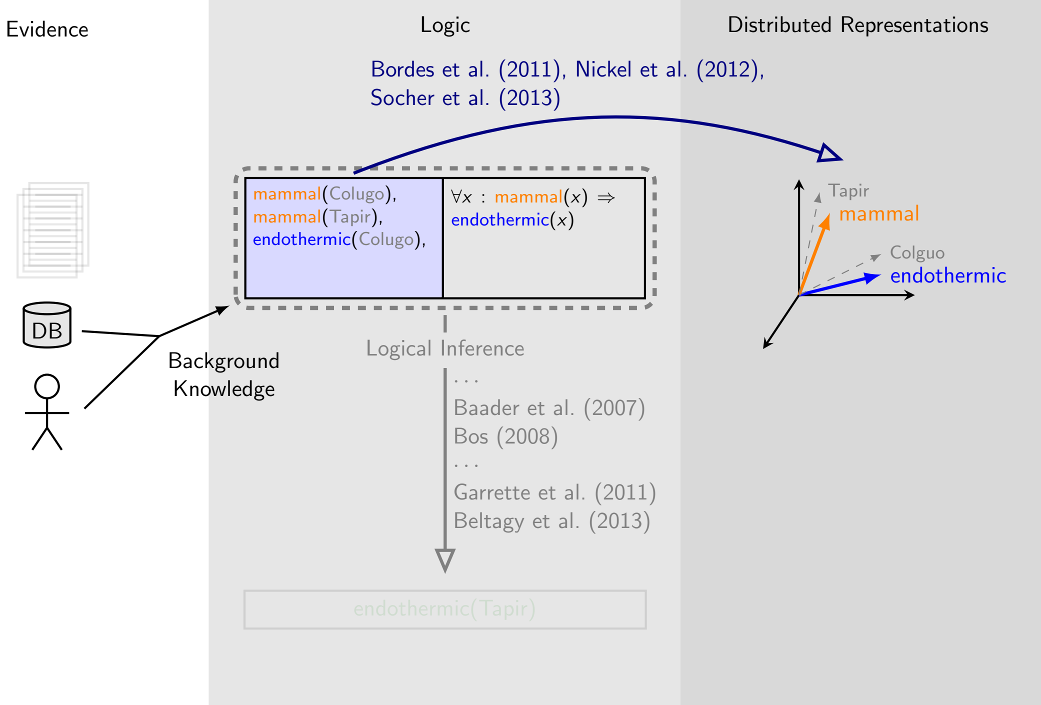


Inference via Distributed Representations



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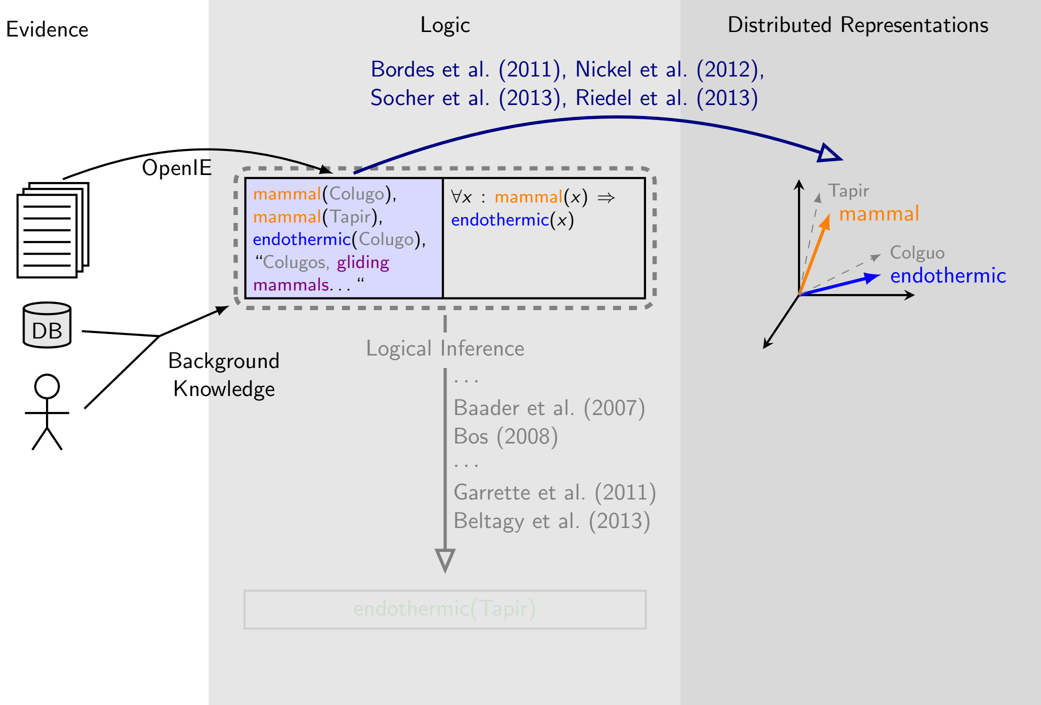

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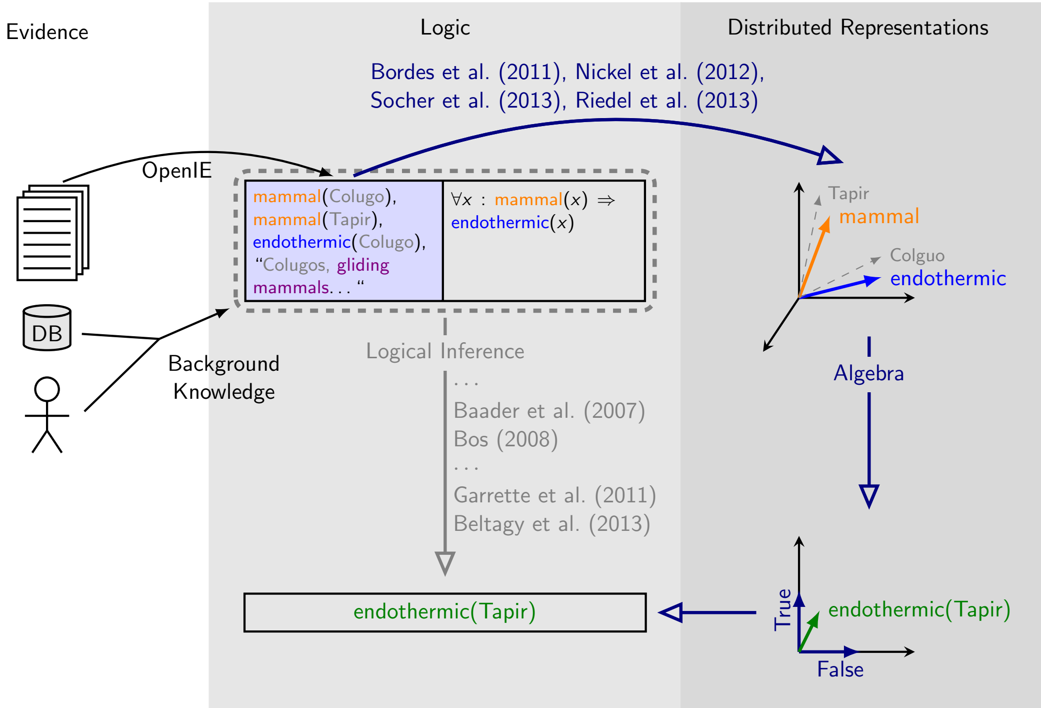
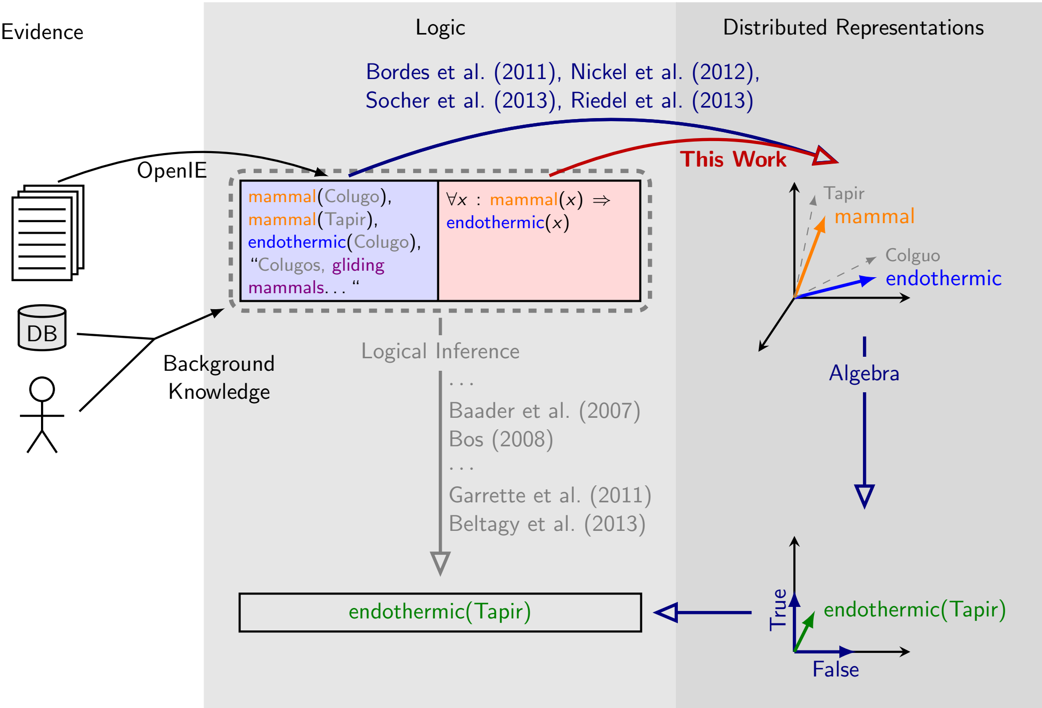


Distributed Representations that Simulate First-order Logical Reasoning



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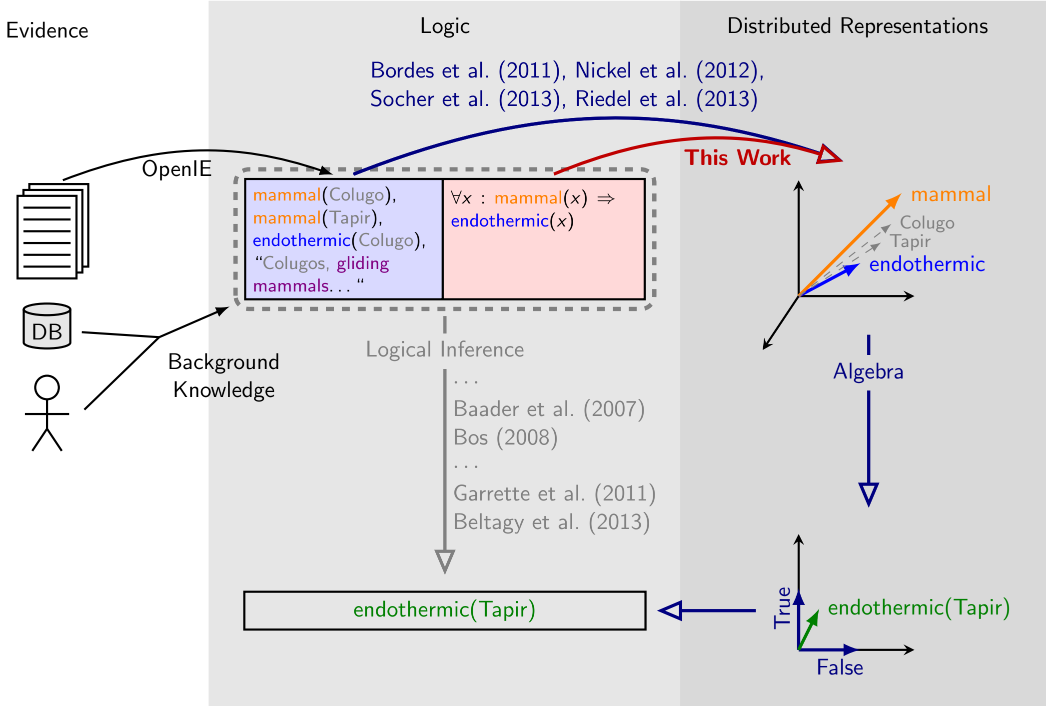

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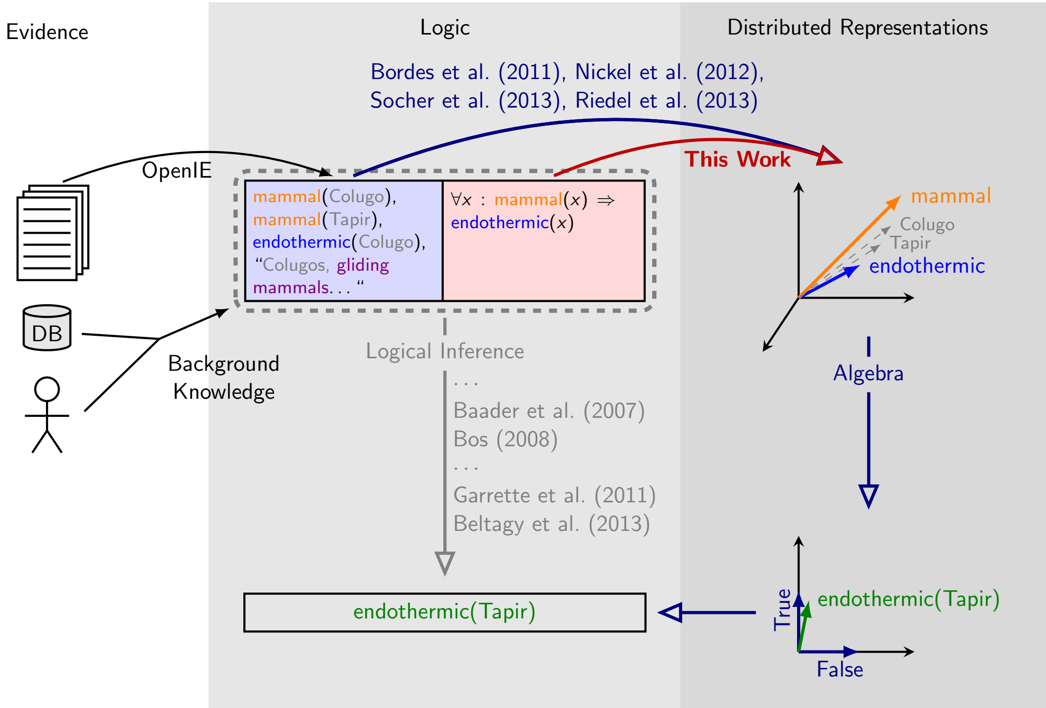

Propositional Logic

Logic	Logical Tensor Calculus (Grefenstette, 2013)
[true]; [false]	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}; \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

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$[\neg\mathcal{A}]$	$[\neg][\mathcal{A}]$

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$[\neg]$; $[\wedge]$; $[\Rightarrow]$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \left[\begin{array}{cc\|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]; \left[\begin{array}{cc\|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{array}\right]$
$[\neg \mathcal{A}]$	$[\neg][\mathcal{A}]$
$[\mathcal{A} \wedge \mathcal{B}]$	$[\wedge] \times_1 [\mathcal{A}] \times_2 [\mathcal{B}]$
$[\mathcal{A} \Rightarrow \mathcal{B}]$	$[\Rightarrow] \times_1 [\mathcal{A}] \times_2 [\mathcal{B}]$

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$[\neg\mathcal{A}]$	$[\neg][\mathcal{A}]$
$[\mathcal{A} \wedge \mathcal{B}]$	$[\wedge] \times_1 [\mathcal{A}] \times_2 [\mathcal{B}]$
$[\mathcal{A} \Rightarrow \mathcal{B}]$	$[\Rightarrow] \times_1 [\mathcal{A}] \times_2 [\mathcal{B}]$
$[\mathcal{A} \wedge \neg\mathcal{B} \Rightarrow \neg\mathcal{C}]$	$[\Rightarrow] \times_1 ([\wedge] \times_1 [\mathcal{A}] \times_2 [\neg][\mathcal{B}]) \times_2 [\neg][\mathcal{C}]$

Constants, Predicates and Quantifiers

	Full-Rank One-Hot Representation (Grefenstette, 2013)
[Colugo]	$\begin{bmatrix} 0 & 0 & 1 & 0 & \cdots & 0 & 0 \end{bmatrix}^\mathsf{T}$

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[MAMMAL]	$\begin{bmatrix} 0 & 0 & 1 & 0 & \cdots & 0 & 1 \\ 1 & 1 & 0 & 1 & \cdots & 1 & 0 \end{bmatrix}$
[MAMMAL(COLUGO)]	[MAMMAL][COLUGO]

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$[\textsc{mammal}]$	$\begin{bmatrix} 0 & 0 & 1 & 0 & \cdots & 0 & 1 \\ 1 & 1 & 0 & 1 & \cdots & 1 & 0 \end{bmatrix}$
$[\textsc{mammal}(\textsc{Colugo})]$	$[\textsc{mammal}][\textsc{Colugo}]$
$[\forall x \in X : F(x)]$	$\begin{cases} [\text{true}] \text{ if } \lvert X \rvert = \\ \quad \lvert \{x \mid F(x) = [\text{true}]\} \rvert \\ [\text{false}] \text{ otherwise} \end{cases}$
$[\exists x \in X : F(x)]$	$\begin{cases} [\text{true}] \text{ if} \\ \quad \lvert \{x \mid F(x) = [\text{true}]\} \rvert > 0 \\ [\text{false}] \text{ otherwise} \end{cases}$

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Example: $Q = \forall x \in X : [\Rightarrow] \times_1 ([\textsc{mammal}][x]) \times_2 ([\textsc{endothermic}][x])$

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	Full-Rank One-Hot Representation (Grefenstette, 2013)	Low-Rank Distributed Representation
$[\textsc{Colugo}]$	$\begin{bmatrix} 0 & 0 & 1 & 0 & \cdots & 0 & 0 \end{bmatrix}^\mathsf{T}$	$\begin{bmatrix} -0.05 & 0.44 & 1.38 \end{bmatrix}^\mathsf{T}$
$[\textsc{mammal}]$	$\begin{bmatrix} 0 & 0 & 1 & 0 & \cdots & 0 & 1 \\ 1 & 1 & 0 & 1 & \cdots & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.56 & 0.24 & 0.63 \\ 0.12 & 0.93 & -0.16 \end{bmatrix}$
$[\textsc{mammal}(\textsc{Colugo})]$	$[\textsc{mammal}][\textsc{Colugo}]$	
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- $\mathcal{E}$: Set of entity (or entity-pair) vectors
- $\mathcal{R}$: Set of relation matrices
- $\mathfrak{K}$: Set of logical formulae Q with training signal $\gamma \in \{[\text{true}], [\text{false}]\}$
  - In previous work this only contained factual statements
  - In addition our objective includes first-order logic formulae!
- $\mathcal{L}$: Loss function, e.g., $\|[Q] - \gamma\|_2$

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Find embeddings for predicates and constants such that true (factual or first-order) formulae evaluate to [true] in the vector space.

$\mathcal{E}$: Set of entity (or entity-pair) vectors

$\mathcal{R}$: Set of relation matrices

$\mathfrak{K}$: Set of logical formulae Q with training signal $\gamma \in \{[\text{true}], [\text{false}]\}$

- In previous work this only contained factual statements
- In addition our objective includes first-order logic formulae!

$\mathcal{L}$: Loss function, e.g., $\|[Q] - \gamma\|_2$

$$\min_{[e]\in\mathcal{E},[r]\in\mathcal{R}} \sum_{(Q,\gamma)\in\mathfrak{K}} \mathcal{L}\left([Q], \gamma\right)$$

Objective

Find embeddings for predicates and constants such that true (factual or first-order) formulae evaluate to [true] in the vector space.

$\mathcal{E}$: Set of entity (or entity-pair) vectors

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$$\min_{[e]\in\mathcal{E},[r]\in\mathcal{R}} \sum_{(Q,\gamma)\in\mathfrak{K}} \mathcal{L}\left([Q], \gamma\right)$$

Gradients: Backpropagation through expression tree of $\mathcal{F}$

Learning: Stochastic Gradient Descent

Example

	MAMMAL	ENDOTHERMIC	VERTERBRATE
CHIMPANZEE	**1.0**	**1.0**	**1.0**
KOALA	**1.0**	**1.0**	**1.0**
COLUGO	**1.0**	?	?
KAGU	?	**1.0**	?
DODO	?	**1.0**	?

Example

	MAMMAL	ENDOTHERMIC	VERTERBRATE
CHIMPANZEE	**1.0**	**1.0**	**1.0**
KOALA	**1.0**	**1.0**	**1.0**
COLUGO	**1.0**	0.1	0.5
KAGU	0.1	**0.9**	0.3
DODO	0.1	**1.0**	0.3

Example

$\forall x \in X : [\Rightarrow] \times_1 ([\textsc{mammal}][x]) \times_2 ([\textsc{endothermic}][x])$

	MAMMAL	ENDOTHERMIC	VERTERBRATE
CHIMPANZEE	**1.0**	**1.0**	**1.0**
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Example

$\forall x \in X : [\Rightarrow] \times_1 ([\textsc{mammal}][x]) \times_2 ([\textsc{endothermic}][x])$

	MAMMAL	ENDOTHERMIC	VERTERBRATE
CHIMPANZEE	**1.0**	**0.9**	**1.0**
KOALA	**1.0**	**0.9**	**1.0**
COLUGO	**0.9**	0.6	0.8
KAGU	0.1	**1.0**	0.2
DODO	0.0	**1.0**	0.1

Example

$\forall x \in X : [\Rightarrow] \times_1 ([\textsc{mammal}][x]) \times_2 ([\textsc{endothermic}][x])$

	MAMMAL	ENDOTHERMIC	VERTERBRATE
CHIMPANZEE	**1.0**	**0.9**	**1.0**
KOALA	**1.0**	**0.9**	**1.0**
COLUGO	**0.9**	(0.1) 0.6	(0.5) 0.8
KAGU	0.1	**1.0**	0.2
DODO	0.0	**1.0**	0.1

Example

$\forall x \in X : [\Rightarrow] \times_1 ([\textsc{mammal}][x]) \times_2 ([\textsc{endothermic}][x])$

	MAMMAL	ENDOTHERMIC	VERTERBRATE
Chimpanzee	**1.0**	**0.9**	**1.0**
Koala	**1.0**	**0.9**	**1.0**
Colugo	**0.9**	(0.1) 0.6	(0.5) 0.8
Kagu	0.1	**1.0**	0.2
Dodo	0.0	**1.0**	0.1

Conclusion and Open Questions

✓ Inject first-order logic into vector spaces

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Conclusion and Open Questions

- ✓ Inject first-order logic into vector spaces
- ✓ Objective that encourages distributed representations to simulate first-order logical reasoning

- ? What are the theoretical limits of embedding logical formulae in vector spaces?
- ? What are efficient ways of injecting quantified formulae without iterating over all elements of a domain?
- ? Can we provide provenance of proofs of answers?

Thank you!

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